Parametric Unity: Beyond the Serial Construct

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Abstract:

The boundary conditions of serialism point toward a new approach to structural organization as described by the *k* construct: a proposed relationship that unites the fundamental parameters of a musical event. This relationship shall be shown to possess unique properties that challenge established precepts of music and open the possibility of a variable musical architecture that can be reinvented for and within each composition. The precision required to implement the *k* construct is most readily attained through the use of computer musical instruments.

"For while it is quite possible to avoid octave-doublings, it is completely impossible to refrain from tone-repetition in such a way that the omission has an effect and can be pointed out—unless one uses some primitive, constantly perceptible, wholly unconcealed, openly visible pre-existing scheme, from which at any time and without any effort on one's own part the fact of non-repetition can be perceived."[1]

Arnold Schoenberg, 1923

1.0 An Empirical Derivation

1.1 Introduction

From a mathematical standpoint, serialism was limited by a finite number of tone row permutations. From an aesthetic standpoint, Schoenberg's "method of composing with twelve tones which are related only with one another"[2] has become a modernist relic. While the method was a great leap forward in terms of the formal elimination of a tonal center, it demanded that each pitch-based decision impart the unyielding momentum of twelve sequential events that could be repeated or occur simultaneously.

The pitches of the tone row were locked in sequence then mirrored in their inversion, retrograde and retrograde inversion forms. Intervallic relationships within the tone row became a code; harmony and counterpoint became functions of the code.[3] If the tone row was unlocked, serialism would either revert to the rules of previous compositional methods or collapse entirely into chaos.

By giving uniform treatment to each pitch class of the present chromatic set—which is uniform by nature—serialism was imbued with the chromatic essence.[4] Therefore, expansion of the set would only constitute an extension of the serial method since the construct of the tone row could be applied to any chromatic base, i.e. number of divisions per octave, provided that all resultant intervals are used.

To advance beyond the serial construct requires the development of a new construct that transcends the boundary conditions of serialism:

(1) If the tone row related sequential events only with one another, the formal elimination of sequential events is required.

(2) If the tone row could be applied to any chromatic base, the formal elimination of the chromatic base is required, as well.

1.2 The k Construct

When composed without repeated tones, each of the twelve events of a tone row contained a pitch ν, an amplitude a and a duration t, (see figure 1).

$$\begin{array}{ccccc} \nu_1 & \nu_2 & \nu_3 & \ldots & \nu_{12} \\ a_1 & a_2 & a_3 & \ldots & a_{12} \\ t_1 & t_2 & t_3 & \ldots & t_{12} \end{array}$$

Figure 1. The Twelve Events of a Tone Row

The tone row was based on a set of fixed intervallic relationships between successive pitches. Such relationships were responsible for the endgame of serialism, wherein a single decision invoked a predetermined sequence of events. While a set of relationships could be formed across various pitches, amplitudes and durations, a sense of audible logic could not be established if such relationships were time-dependent. What remains is to form a construct based on a single relationship within an isolated event. Such an event, denoted by *x*, contains a single pitch, amplitude and duration, (see figure 2).

$$\begin{array}{c} v_x \\ a_x \\ t_x \end{array}$$

Figure 2. An Isolated Event

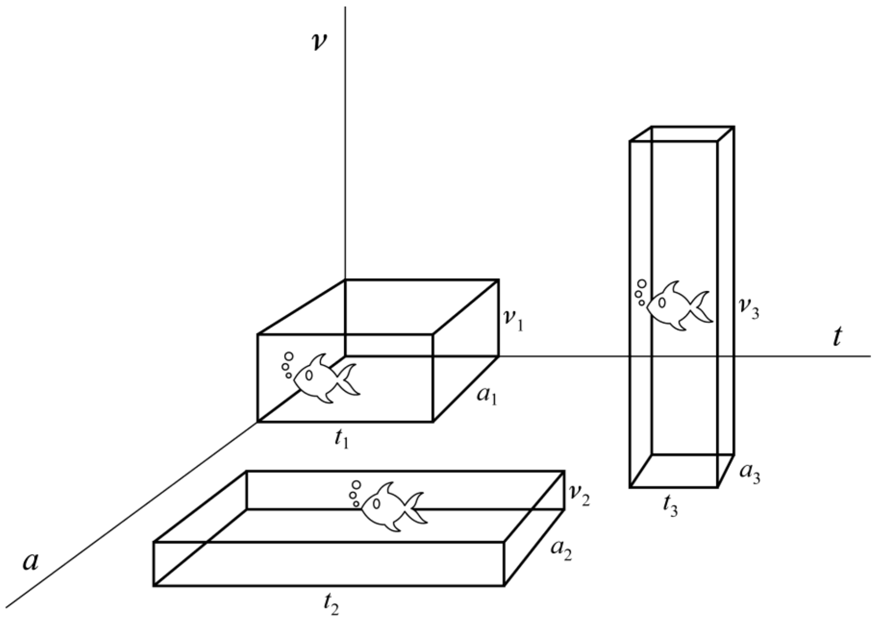
In the absence of any rules governing relationships between diverse musical parameters, a basic construct can be formed via the introduction of an arbitrary constant *k*, (see figure 3).

$$k = v_x \, a_x \, t_x$$

Figure 3. The *k* Construct

By applying this construct to all events within a composition, any two parameters of any event can have any value provided that the relationship is satisfied. This relationship, referred to as *the k construct*, shall be shown to possess unique properties that challenge established precepts of music and enable the boundary conditions of serialism to be transcended.

1.3 A Conceptual Model of The k Construct



$$k = \nu_1 a_1 t_1 = \nu_2 a_2 t_2 = \nu_3 a_3 t_3$$

Figure 4. Musical Events Containing Equal Geometric Volumes

Using the k construct, each event within a composition can be described as a fish tank that has been completely filled with water, (see figure 4). The height, width and length of each tank correspond to the pitch ν, amplitude a and duration t of each event. The dimensions of each tank can vary, however, for a given value of k, all tanks contain the same amount of water.

2.0 A Theoretical Basis

2.1 Compositional Structure

ν_1 ↔	ν_2 ↔	ν_3 ↔	ν_4 ↔	ν_5 ↔	ν_6 ↔	ν_7 ↔	ν_8 ↔	ν_9 ↔	ν_{10} ↔	ν_{11} ↔	ν_{12}
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}

Figure 5. The Parametric Structure of the Tone Row

Figure 6. The Atomic Structure of the Tone Row

(Anton Webern, Concerto, Op. 24, No. I, 1, Basic Set.)

The structure of a composition can be divided into two parts: the *parametric structure*, which defines the nature of relationships among designated parameters and the *atomic structure*, which defines the smallest building block of the overall composition. As the term suggests, the parametric structure of the “tone row” was confined to localized relationships between successive pitches, (see figure 5). The atomic structure was an ordered set of twelve events that utilized each pitch class of the chromatic set only once, (see figure 6).

ν_1	ν_2	ν_3	ν_4	ν_5	ν_6	ν_7	ν_8	ν_9	ν_{10}	ν_{11}	ν_{12}
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}

Figure 7. The Parametric Structure of an Isolated Event

$$k = v_x \, a_x \, t_x$$

Figure 8. The Atomic Structure of an Isolated Event

The key to unlocking the tone row without theoretic reversion or collapse lies in expanding the parametric structure of each event to include amplitude and duration. This expansion facilitates a shift from a set of fixed intervallic relationships between successive pitches in the horizontal plane to one variable parametric relationship within each isolated event in the vertical plane, (see figure 7). Note: This approach is not the same as "Total" or "Integral" serialism, which applied sets of fixed intervallic relationships to diverse musical parameters.

The atomic structure of an isolated event—the element needed to tie one event to another—is given by *k*, (see figure 8). While the tone row provided a cohesive lattice at the expense of required cyclical repetition, *k* provides a single point of reference: a nucleus that is independent of the sequential order of events. By expanding the parametric structure of isolated events and thereby contracting the atomic structure from twelve specifications for pitch to one specification for *k*, each event can embody the overall structure of the entire composition.

This change in perspective is analogous to the shift from Bach's modulating tonal center to Schoenberg's tone row. By expanding tonal structure to include the uniform treatment of all pitch classes, Schoenberg collapsed all of tonality into a single event related only with its immediate neighbors. By expanding the parametric structure of a single event to include the uniform treatment of all musical parameters, the *k* construct collapses the relationship between immediate neighbors, removing the event from temporal context.

Thus, the expansion of parametric structure and the contraction of atomic structure provide the means to transcend the first boundary condition of serialism by eliminating the need for sequential events. This is not to say that an event cannot be recontextualized within a temporal framework in order to serve a cognitive function; this is only to say that an affinity toward its own intrinsic parameters is given priority. While the elimination of sequential events is achieved through Cartesian principles, the elimination of the chromatic base requires a more thorough historical analysis.

2.2 The Architecture of Western Music

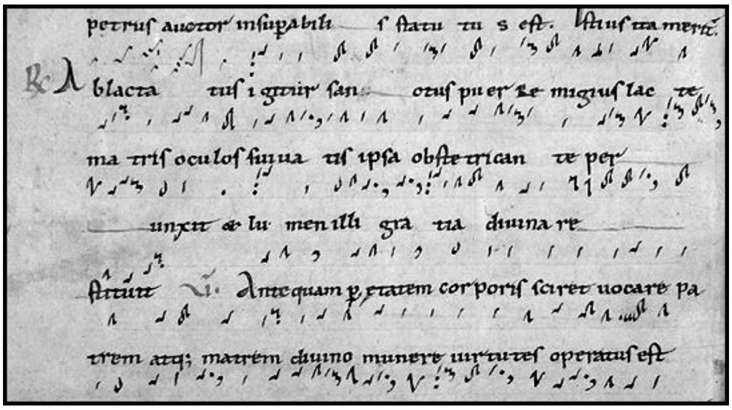


Figure 9. Cheironomic Neumes of the Middle Ages [5]

The history of musical notation describes how the architecture of Western music has evolved in terms of parametric differentiation. While the subject is one of great debate, musical events were first differentiated by absolute pitch in the early Middle Ages, followed by the modification of Gregorian neumes to signify relative time and the Baroque introduction of relative dynamics. Over the centuries, each parameter would evolve its own highly specialized set of operations and refinements.

Such expanding differentiation suggests the existence of a prior undifferentiated state whose traces are visible in the cheironomic neumes of the Middle Ages, (see figure 9). By mixing symbols for diverse musical and linguistic attributes into a single continuous stream, this form of notation gave uniform treatment to: 1) pitch inflection in terms of upward or downward movement, 2) amplitude inflection in terms of phonetic accentuation, and 3) temporal inflection in terms of grammatical punctuation.[6]

In physical terms, cheironomic neumes were named for the hand gestures used to direct a vocal performance. However, on a deeper level, the single continuous (and undulating) stream gave symbolic form to the breath—which gauged choral melodies sung in unison, rise-and-fall amplitudes, and isometric durations to the span of a single exhale. While musical parameters can be easily differentiated in a written context, in a biological context they cannot.

By treating pitch, amplitude and duration as manifestations of a single entity, an arc begins to form between Early Music and the parametrically undifferentiated state described by the *k* construct. In between lies a dialectic, first dominated by musical architecture which divided and mapped the sonic landscape, then dominated by compositional structure which gradually filled the terrain as composers made ever-increasing use of the full chromatic set.

2.3 The Convergence and Singularity of Pitch

In the Western tradition, the *convergence* or union of compositional structure and musical architecture was driven by the emancipation of dissonance and resulted in a mirror between the tone row and the chromatic set. The parametric structure of the tone row reflected the very nature of movement by intervals, with each step dependent only upon the previous location. The atomic structure of the tone row reflected the very nature of division by intervals, accounting for the full and unweighted palette of tones from which all of Western music is derived.

The effect of this convergence was *Schoenberg's paradox* wherein the complete localization of pitch-based relationships would allow no further contraction of parametric structure between events and any architectural expansion of the chromatic set would only be assimilated by atomic structure in the form of an expanded tone row. While not overtly stated by Schoenberg, the presence of an underlying paradox is consistent with his droll prophecy made in 1921 to "ensure the supremacy of German music for the next hundred years."[7]

As a corollary to the elimination of sequential events, the paradox is resolved when the pitch of an isolated event engages its own intrinsic amplitude and duration. Free to disengage from harmonic and contrapuntal interaction, the parameter enters a state of *singularity* wherein the relationship between immediate neighbors contracts to zero and its values may be chosen from a continuous range, in this case, without geometric derivation or temperament.

The historical re-engagement of pitch to amplitude and duration provides the means to transcend the second boundary condition of serialism by eliminating the need for the chromatic base. Once again, this is not to say that an event cannot be recontextualized within a chromatic framework in order to serve a cognitive function; this is only to say that an affinity toward its own intrinsic parameters is given priority.

2.4 Parametric Unity

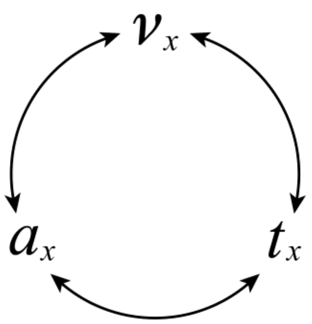


Figure 10. The Imploded Exchange of Parametric Unity

The parameters of amplitude and duration have no analog to the convergence of pitch in the Western tradition, but the phenomenon may exist in the music of other cultures. Even without direct evidence, one can imagine three unique musical traditions: a tonal tradition based on pitch, a dynamic tradition based on amplitude, and a rhythmic tradition based on duration, wherein the respective compositional structures and architectures have converged and the defining parameter of each tradition has entered a state of singularity. Equidistant from each singularity lies a point of structural equivalence: the point of *parametric unity*, where the relationship between the immediate neighbors of any parameter contracts to zero and the values of any parameter may be chosen from a continuous range without any form of intervallic differentiation or adjustment.

If the architecture of a musical tradition represents its compositional potential and convergence represents the fulfillment of its potential, then singularity represents both its origin and its ultimate fate. Poised in a space between the death and birth of musical traditions, parametric unity belongs to a sonic order that is free of historical bias. In a state of parametric unity given by $k = \nu_x\ a_x\ t_x$, the fundamental parameters of a musical event participate in an imploded exchange, (see figure 10) where pitch can become amplitude and amplitude can become duration, transforming in endless permutation, oblivious of every other event. Therefore, k is defined as the determinant of parametric unity following the convergence and singularity of all pitch, amplitude and duration-based compositional structures with their respective architectures.

2.5 A Conceptual Model of Parametric Unity

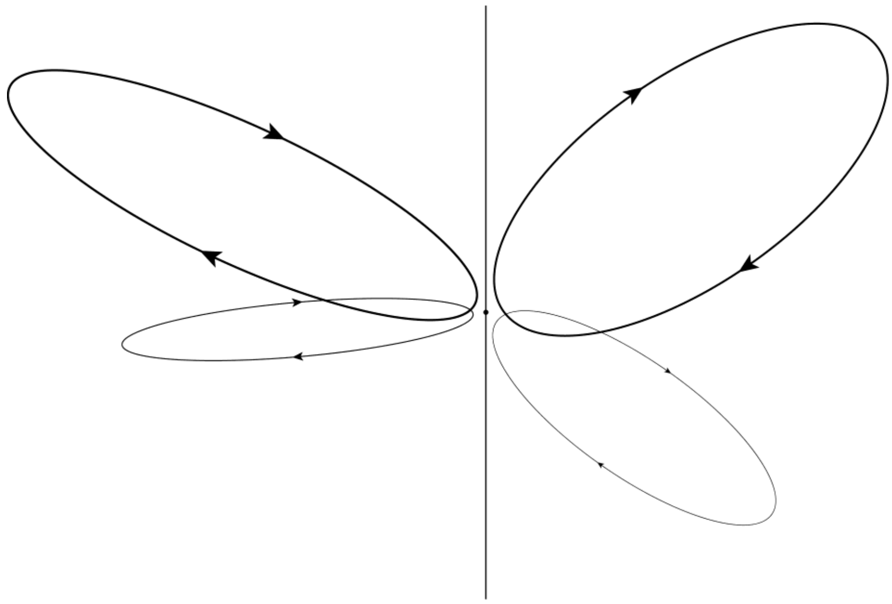


Figure 11. Musical Traditions Surrounding the Point of Parametric Unity

(The arrows indicate the direction of time.)

Since the point of parametric unity lies in a space between the death and birth of musical traditions that is free of historical bias, each tradition can be described as an elliptical path that begins and ends in closest proximity to this point, (see figure 11). The dialectic between musical architecture and compositional structure that forms the arc of each path begins and ends with the defining parameter in a state of singularity.

3.0 Musical Application

3.1 Applying The k Construct

The first step in applying the *k* construct is to choose the units of measure for pitch, amplitude and duration. Each parameter can use scientific or musical units: e.g. pitch can be measured in terms of frequency (Hz) or pitch class, amplitude in decibels (dB) or relative dynamics, and duration in seconds (s) or metered time. In this application, units that reflect the smallest variances over their respective ranges produce events most closely associated with natural music.

For example, frequency doubles over a single octave while pitch class increases by a factor of twelve, and decibel levels increase logarithmically while relative dynamics are linear. Both seconds and metered time are linear and can be interchanged via the tempo. Therefore, frequency shall be chosen for pitch, relative dynamics for amplitude, and seconds for duration.

The second step is to choose an arbitrary set of ranges for each parameter, (see figure 12).

$$100.00 \text{ Hz} \leq \nu_x \leq 4{,}000.00 \text{ Hz}$$
$$1.00 \leq a_x \leq 10.00$$
$$0.25 \text{ s} \leq t_x \leq 4.00 \text{ s}$$

Figure 12. An Arbitrary Set of Ranges for Each Parameter

From the chosen set of ranges, the maximum and minimum values of *k* can be determined, (see figure 13).

$$k_{max} = (4{,}000.00)\ (10.00)\ (4.00) = 160{,}000.00$$

$$k_{min} = (100.00)\ (1.00)\ (0.25) = 25.00$$

Figure 13. The Maximum and Minimum Values of *k*

The third step is to choose a value for *k* between k_{max} and k_{min}. Any value may be chosen, however, the optimum value of *k* that will normalize the diverse ranges of pitches, amplitudes and durations shall be defined as the geometric mean, (see figure 14).

$$k_{opt} = [(k_{max})\ (k_{min})]^{1/2}$$

$$k_{opt} = [(160{,}000.00)\ (25.00)]^{1/2} = 2{,}000.00$$

Figure 14. The Optimum Value of *k*

Note: The precision required to implement the *k* construct is most readily attained through the use of computer musical instruments.

3.2 Variations on the Basic Set from Webern's Concerto, Op. 24

The process of unlocking the tone row shall be demonstrated by applying the *k* construct to the basic set from Webern's Concerto, Op. 24. The series of variations that follow shall transform the basic set until all parameters have entered a state of singularity. In order to make the transformation gradual, the series shall trace the evolution of Western parametric differentiation in reverse, from amplitude to duration and finally to pitch.

3.2.1 Analysis of the Basic Set

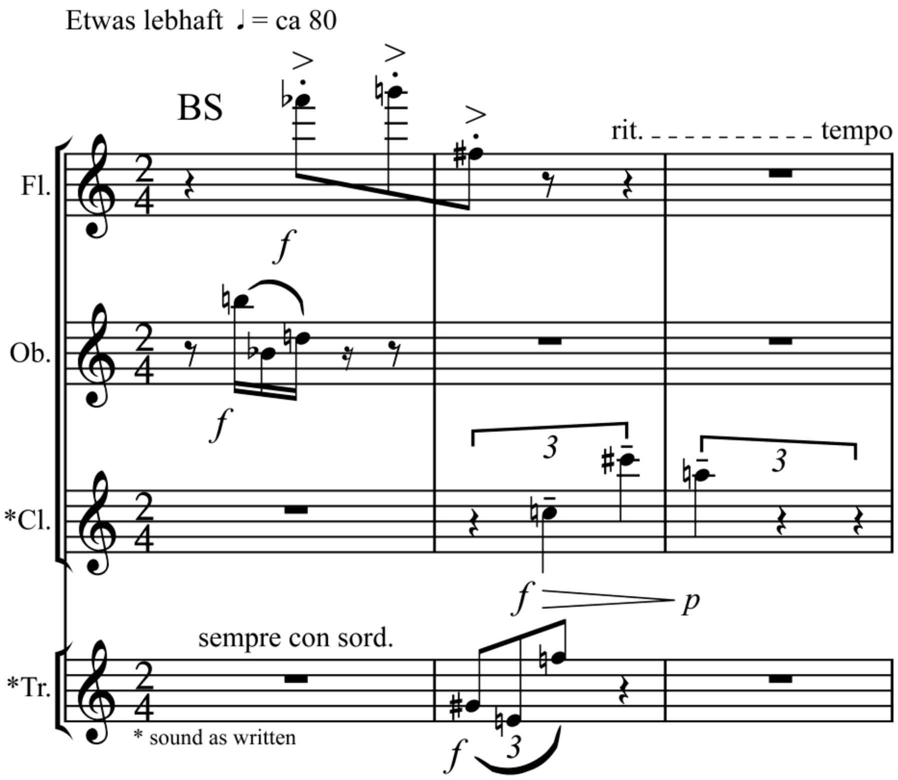


Figure 15. Anton Webern, Concerto, Op. 24, No. I, 1, Basic Set [8]

x	Pitch v_x - Op. 24		Amplitude a_x - Op. 24		Duration t_x - Op. 24	
Event Number	Pitch Class	Absolute Pitch (Hz)	Relative Dynamics	Relative Scale 1.00 - 8.00	Relative Time	Absolute Time @ 80 bpm
1	e	493.88	forte	6.00	1/16	0.1875
2	t	233.08	forte	6.00	1/16	0.1875
3	2	293.66	forte	6.00	1/16	0.1875
4	3	622.25	forte	6.00	1/8 staccato	0.1875
5	7	783.99	forte	6.00	1/8 staccato	0.1875
6	6	369.99	forte	6.00	1/8 staccato	0.1875
7	8	207.65	forte	6.00	1/8 triplet	0.2500
8	4	164.81	forte	6.00	1/8 triplet	0.2500
9	5	349.23	forte	6.00	1/8 triplet	0.2500
10	0	261.63	forte	6.00	1/4 triplet	0.5000
11	1	554.37	*mf* / *mp*	4.50	1/4 triplet	0.5000
12	9	440.00	piano	3.00	1/4 triplet	0.5000

Figure 16. The Individual Events of the Basic Set

The basic set shown in figure 15 has been converted into numeric values in figure 16. Each pitch has been converted into a value measured in Hertz, each relative dynamic has been converted into a value between 1.00 and 8.00 (where *ppp* = 1.00, *pp* = 2.00, *p* = 3.00, *mp* = 4.00, *mf* = 5.00, *f* = 6.00, *ff* =

7.00, *fff* = 8.00), and each duration has been converted into a value measured in seconds. Based on these values, the set of ranges for the basic set is shown in figure 17.

$$164.81\text{ Hz} \leq \nu_x \leq 783.99\text{ Hz}$$
$$1.00 \leq a_x \leq 8.00$$
$$0.1875\text{ s} \leq t_x \leq 0.5000\text{ s}$$

Figure 17. The Set of Ranges for the Basic Set

From the set of ranges for the basic set, the maximum and minimum values of k are shown in figure 18.

$$k_{max} = (783.99)\ (8.00)\ (0.5000) = 3135.96$$

$$k_{min} = (164.81)\ (1.00)\ (0.1875) = 30.90$$

Figure 18. The Maximum and Minimum Values of k for the Basic Set

From the maximum and minimum values of k, the optimum value of k is shown in figure 19.

$$k_{opt} = [(k_{max})\ (k_{min})]^{1/2}$$

$$k_{opt} = [(3135.96)\ (30.90)]^{1/2} = 311.30$$

Figure 19. The Optimum Value of k for the Basic Set

3.2.2 An Amplitude-Dependent Variation

Using the pitches and durations from the basic set and the optimum value of k, a new set of amplitudes is shown in figure 20 and figure 21.

Figure 20. An Amplitude-Dependent Variation

x	*k*	Pitch ν_x Op. 24	Duration t_x Op. 24	Amplitude a_x @ k_{opt}
Event Number	Optimum k_{opt}	Absolute Pitch (Hz)	Absolute Time @ 80 bpm	Relative Scale 1.00 - 8.00
1	311.30	493.88	0.1875	3.36
2	311.30	233.08	0.1875	7.12
3	311.30	293.66	0.1875	5.65
4	311.30	622.25	0.1875	2.67
5	311.30	783.99	0.1875	2.12
6	311.30	369.99	0.1875	4.49
7	311.30	207.65	0.2500	6.00
8	311.30	164.81	0.2500	7.56
9	311.30	349.23	0.2500	3.57
10	311.30	261.63	0.5000	2.38
11	311.30	554.37	0.5000	1.12
12	311.30	440.00	0.5000	1.41

Figure 21. The Individual Events of the Amplitude-Dependent Variation

The amplitude of each event is equal to the optimum value of *k* divided by the product of its pitch and duration. The calculations for the first three events are shown in figure 22.

$$a_x = k_{opt} / [(v_x)\ (t_x)]$$

$$a_1 = 311.30 / [(493.88)\ (0.1875)] = 3.36$$

$$a_2 = 311.30 / [(233.08)\ (0.1875)] = 7.12$$

$$a_3 = 311.30 / [(293.66)\ (0.1875)] = 5.65$$

Figure 22. The Amplitudes for Events $x = 1$, $x = 2$, and $x = 3$

The compositional structure (1) and musical architecture (2) of the amplitude-dependent variation appear as follows.

From the perspective of the Western tradition:

(1) The sequential order of the tone row and its corresponding durations have been retained. Conversely, the amplitude of each event derives from its pitch and duration, and therefore disengages from the amplitudes of its immediate neighbors.

(2) The predefined operations and refinements of pitch in terms of chromatic division and temperament, and duration in terms of geometric division and staccato, have also been retained. Conversely, the continuous range of amplitudes originates from a separate, real or imaginary tradition that has undergone a process of convergence and singularity.

From the perspective of parametric unity (which is, by definition, tradition less):

(1) The fact that the variation replicates the sequential order of the tone row and its corresponding durations is purely coincidental since all sequential relationships arise from the parametric core of each event in isolation.

(2) Since each parameter has a continuous range, the architecture is completely variable and can assume any shape. In this case, the pitches and durations exhibit the characteristics of a superimposed Western set of operations and refinements *ex post facto* while the amplitudes serve to maintain the parametric balance of each event.

3.2.3 A Duration-Dependent Variation

Using the pitches from the basic set, amplitudes chosen at random, and the optimum value of *k*, a new set of durations is shown in figure 23. Note: The use of random numbers serves to demonstrate that any value within the chosen range is feasible. All random numbers were generated by the sampling of atmospheric noise.

x	k	Pitch v_x Op. 24	Amplitude a_x Random	Duration t_x @ k_{opt}
Event Number	Optimum k_{opt}	Absolute Pitch (Hz)	Relative Scale 1.00 - 8.00	Absolute Time (s)
1	311.30	493.88	6.35	* 0.0993
2	311.30	233.08	7.06	0.1892
3	311.30	293.66	4.26	0.2488
4	311.30	622.25	2.22	0.2254
5	311.30	783.99	4.88	* 0.0814 †
6	311.30	369.99	6.80	* 0.1237
7	311.30	207.65	5.56	0.2696
8	311.30	164.81	6.91	0.2733
9	311.30	349.23	6.66	* 0.1338
10	311.30	261.63	4.38	0.2717
11	311.30	554.37	1.76	0.3191
12	311.30	440.00	4.16	* 0.1701

* Values under range
† Minimum original duration (*original* t_{min})

Figure 23. The Individual Events of a Duration-Dependent Variation for the Optimum Value of k

In this case, many of the durations fall below the lower limit of the range chosen for the parameter. While maintaining values within a chosen set of ranges may be impossible in the *post hoc* analysis of an existing score, any parameter can be scaled to ensure that no value falls either above or below a chosen limit.

To limit the minimum scaled duration (*scaled* t_{min}) to 0.1875 seconds, each scaled duration (*scaled* t_x) is equal to the minimum scaled duration multiplied by the ratio of the original duration (*original* t_x) to the minimum original duration (*original* t_{min}). The calculations for the first three events are shown in figure 24.

$$\textit{scaled } t_x = \textit{scaled } t_{min} [(\textit{original } t_x) / (\textit{original } t_{min})]$$

$$\textit{scaled } t_1 = 0.1875 [(0.0993) / (0.0814)] = 0.2287$$

$$\textit{scaled } t_2 = 0.1875 [(0.1892) / (0.0814)] = 0.4359$$

$$\textit{scaled } t_3 = 0.1875 [(0.2488) / (0.0814)] = 0.5734$$

Figure 24. The Scaled Durations for Events $x = 1$, $x = 2$, and $x = 3$

Note: Durations are scaled to a maximum limit by substituting t_{max} for t_{min}, pitches are scaled to a maximum or minimum limit by substituting v for t, and amplitudes are scaled to a maximum or minimum limit by substituting a for t.

The scaling of any parameter will require that the value of k be revised accordingly, as shown in figure 25. Since the original durations were related by a single optimum value of k, the scaled durations are related by a single revised value of k.

$$k_{rev} = (\nu_x)\ (a_x)\ (scaled\ t_x)$$

For $x = 1$: $k_{rev} = (493.88)\ (6.35)\ (0.2287) = 717.35$

For $x = 2$: $k_{rev} = (233.08)\ (7.06)\ (0.4359) = 717.35$

For $x = 3$: $k_{rev} = (293.66)\ (4.26)\ (0.5734) = 717.35$

Figure 25. The Revised Value of *k* for Events $x = 1$, $x = 2$, and $x = 3$

By scaling the durations and revising the value of *k*, the resulting duration-dependent variation is shown in figure 26 and figure 27.

x	Pitch ν_x Op. 24	Amplitude a_x Random	Duration t_x @ k_{rev}	k
Event Number	Absolute Pitch (Hz)	Relative Scale 1.00 - 8.00	Absolute Time (s)	Revised k_{rev}
1	493.88	6.35	0.2287	717.35
2	233.08	7.06	0.4359	717.35
3	293.66	4.26	0.5734	717.35
4	622.25	2.22	0.5193	717.35
5	783.99	4.88	0.1875	717.35
6	369.99	6.80	0.2851	717.35
7	207.65	5.56	0.6213	717.35
8	164.81	6.91	0.6299	717.35
9	349.23	6.66	0.3084	717.35
10	261.63	4.38	0.6260	717.35
11	554.37	1.76	0.7352	717.35
12	440.00	4.16	0.3919	717.35

Figure 26. The Individual Events of a Duration-Dependent Variation
for a Revised Value of *k*

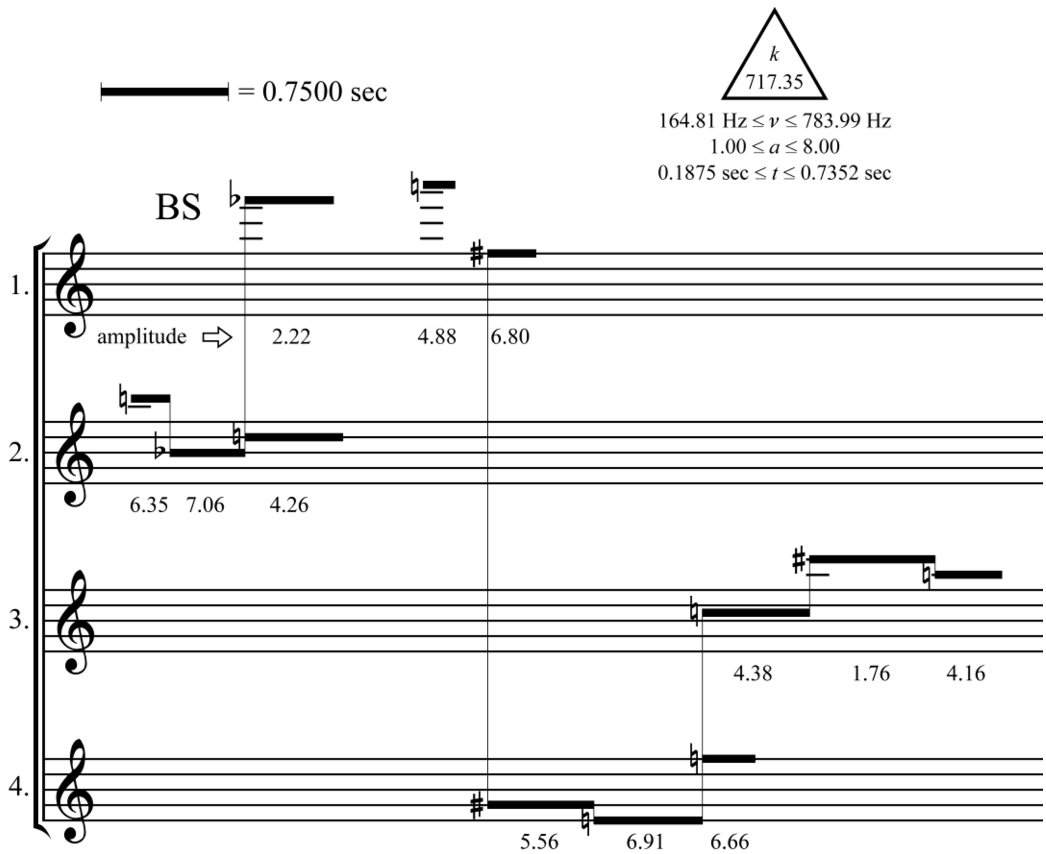


Figure 27. A Duration-Dependent Variation

Since the durations have a continuous range, each event in the score is represented by a bar whose length corresponds to its duration. Note: There is no need for the score to indicate whether *k* is optimum or revised, or whether a parameter is original or scaled, since the *k* construct remains valid in all cases. For the purpose of demonstration, events 4, 5, and 6 have been assigned their full durations in order to maintain the parametric balance of each event, however, rests equivalent to the full durations have been inserted after events 4 and 5 in order to maintain the character of the staccato designation. As an alternative, halved durations with doubled amplitudes followed by halved rests could have been used if both *a* and *t* were scaled and *k* was revised accordingly.

The compositional structure (1) and musical architecture (2) of the duration-dependent variation appear as follows.

From the perspective of the Western tradition:

(1) The sequential order of the tone row has been retained. Conversely, each amplitude and duration disengages from its respective neighbors.

(2) The operations and refinements of pitch have also been retained. Conversely, the continuous ranges of amplitudes and durations originate from either two separate, real or imaginary traditions that have each undergone a process of convergence and singularity, or one united tradition that has undergone a similar process.

From the perspective of parametric unity:

(1) The fact that the variation replicates the sequential order of the tone row is purely coincidental since all sequential relationships arise from the parametric core of each event in isolation.

(2) The pitches exhibit the characteristics of a superimposed Western set of operations and refinements *ex post facto* while the amplitudes and durations serve to maintain the parametric balance of each event.

3.2.4 The Tone Row Unlocked

Since the architecture of a parametrically united event can conform to any pitch, a transpositional equivalence exists between any member of the chromatic set and any other frequency provided that a corresponding change in amplitude, duration or both is achievable in accordance with the *k* construct. Thus, for any tone row composed of parametrically united events, each pitch is a single manifestation of a continuous range of pitches that can be interposed and substituted at will.

Any octave transposition within the tone row would fully preserve its compositional structure while any non-octave chromatic transposition would collapse its serial atomic structure alone. However, any non-chromatic transposition unassociated with temperament or discordant behavior would create an architectural discontinuity that collapses both the serial atomic structure by eliminating the chromatic base, and—without any means of relating successive pitches—the serial parametric structure by eliminating sequential events.

Therefore, with a deliberate yet infinitesimal shift in the frequency of any event accompanied by its parametric counterbalance, as shown in figure 28 and figure 29, the tone row is unlocked. While the passage appears virtually unchanged, a fundamental change has occurred in its musical architecture. Each pitch, while seemingly part of the discrete chromatic set, is now part of a continuum and each event, while seemingly interrelated by pitch, is now only contiguous or non-contiguous. The unaltered pitches exhibit the characteristics of a serial region encased within a superseding order.

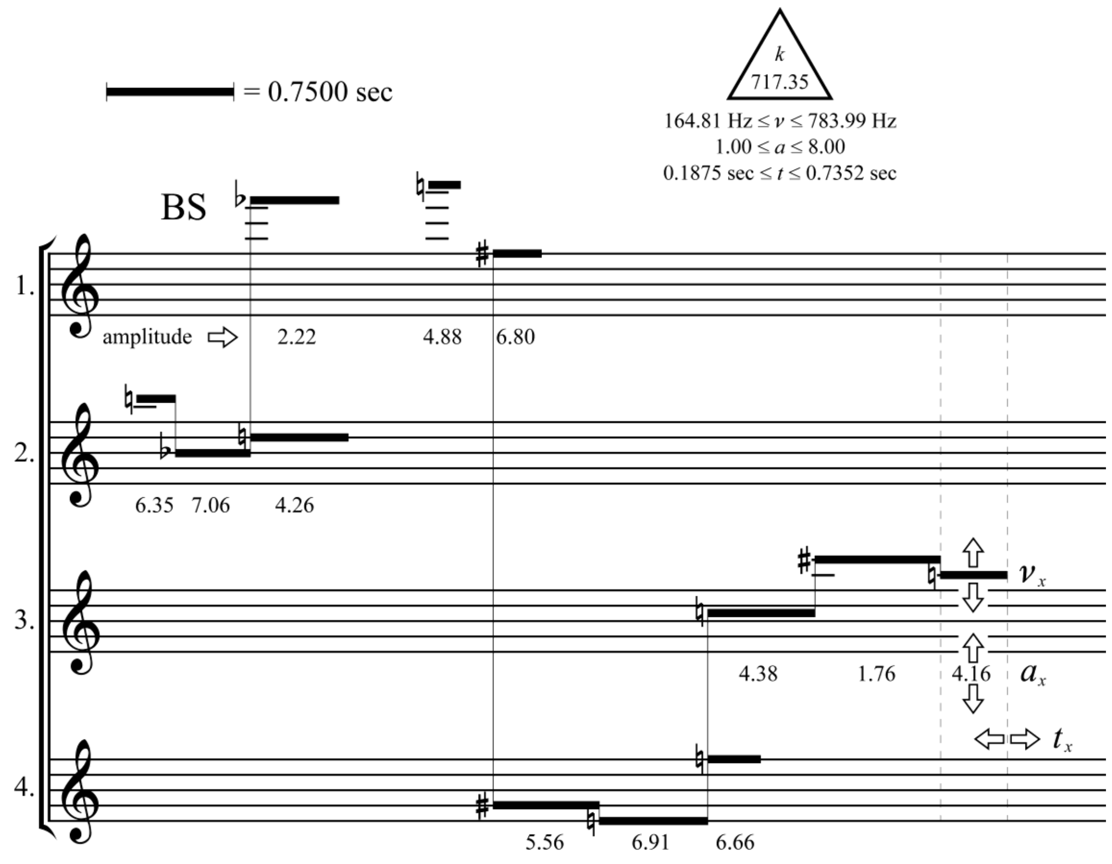


Figure 28. The Tone Row Unlocked

x	k	Pitch ν_x	Amplitude a_x	Duration t_x
Event Number	Revised k_{rev}	Absolute Pitch (Hz)	Relative Scale 1.00 - 8.00	Absolute Time (s)
12	717.35	440.01	4.15	0.3928
12	717.35	440.00	4.16	0.3919
12	717.35	439.99	4.17	0.3910

Figure 29. The Non-Chromatic Transposition of Event x = 12 (A = 440)

While a pitch-dependent variation of the basic set is antithetical to the serial construct, one can be formed by applying the *k* construct. Using amplitudes chosen at random, the identical durations from figure 26 for the purpose of demonstration, and the revised value of *k*, a new set of pitches is shown in figure 30 and figure 31.

x	k	Amplitude a_x Random	Duration t_x Figure 26	Pitch ν_x @ k_{rev}
Event Number	Revised k_{rev}	Relative Scale 1.00 - 8.00	Absolute Time (s)	Absolute Pitch (Hz)
1	717.35	6.94	0.2287	451.89
2	717.35	2.60	0.4359	633.67
3	717.35	2.30	0.5734	544.26
4	717.35	3.18	0.5193	434.44
5	717.35	5.77	0.1875	663.09
6	717.35	7.21	0.2851	348.91
7	717.35	3.28	0.6213	352.48
8	717.35	1.95	0.6299	583.82
9	717.35	7.65	0.3084	303.86
10	717.35	2.82	0.6260	406.34
11	717.35	3.10	0.7352	314.50
12	717.35	3.59	0.3919	509.30

Figure 30. The Individual Events of a Pitch-Dependent Variation for a Revised Value of *k*

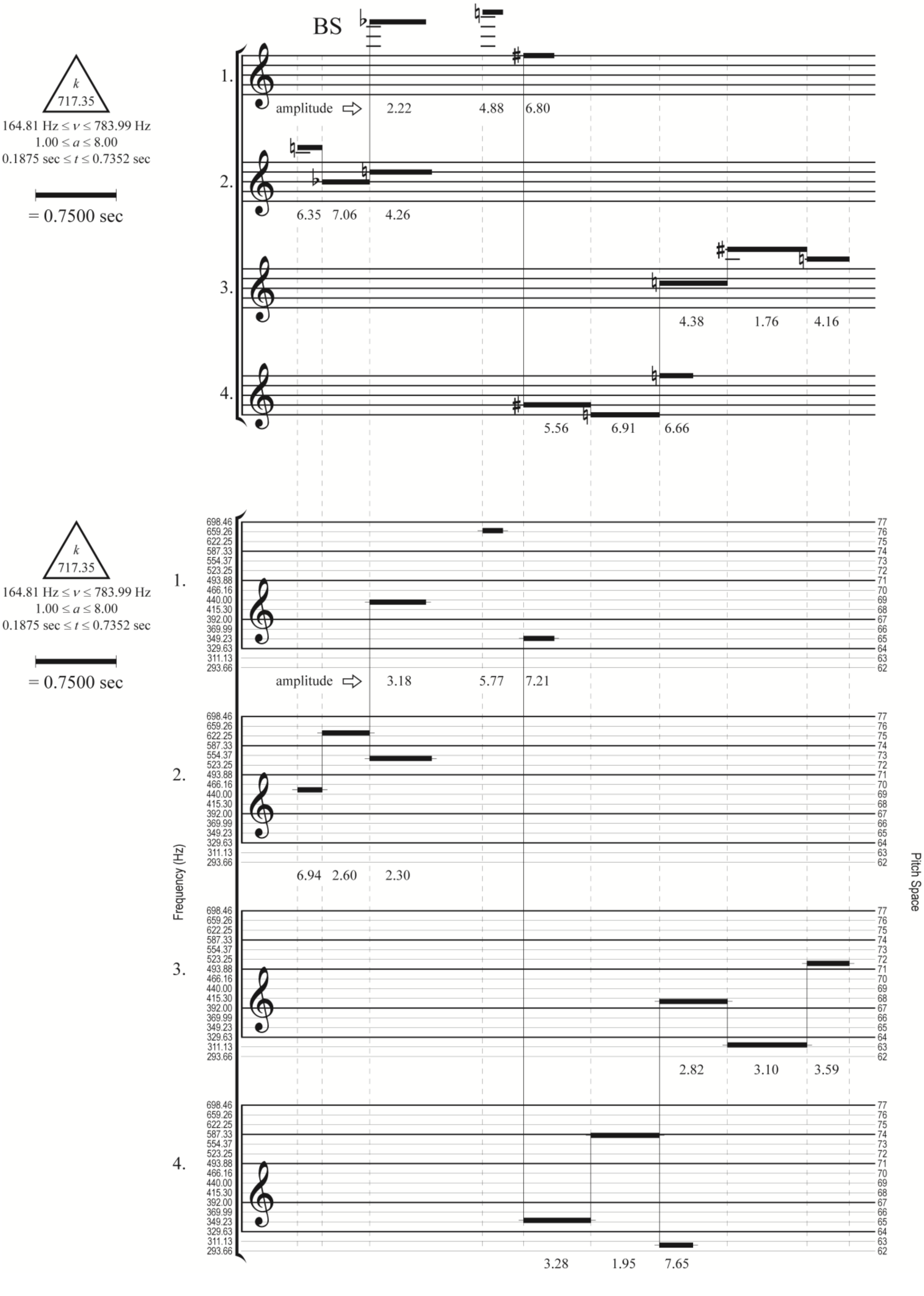


Figure 31. A Pitch-Dependent Variation

3.2.5 The Departure of Parametric Unity

Without further need for the chromatic set, the last remains of the architecture underlying the serial construct—namely the requirement of a basic set consisting of twelve events—can be abandoned. Thus, an entirely new passage can be formed for any number of events in accordance with the *k* construct, as shown in figure 32, figure 33, and figure 34, and what began as a rudimentary amplitude variation of a basic set culminates in an independent musical paradigm.

x	*k*	Pitch ν_x	Amplitude a_x	Duration t_x
Event Number	Arbitrary *k*	Absolute Pitch (Hz)	Relative Scale 1.00 - 8.00 *	Absolute Time (s)
1	1000.00	154.67	2.25	2.8735
2	1000.00	615.91	4.63	0.3507
3	1000.00	312.23	3.32	0.9647
4	1000.00	434.24	1.44	1.5992
5	1000.00	187.06	3.85	1.3885
6	1000.00	576.24	1.00	1.7354
7	1000.00	122.41	2.64	3.0944
8	1000.00	414.61	4.46	0.5408
9	1000.00	252.55	4.36	0.9082

* Range restricted to 1.00 - 5.00

Figure 32. Individual Events in a State of Parametric Unity

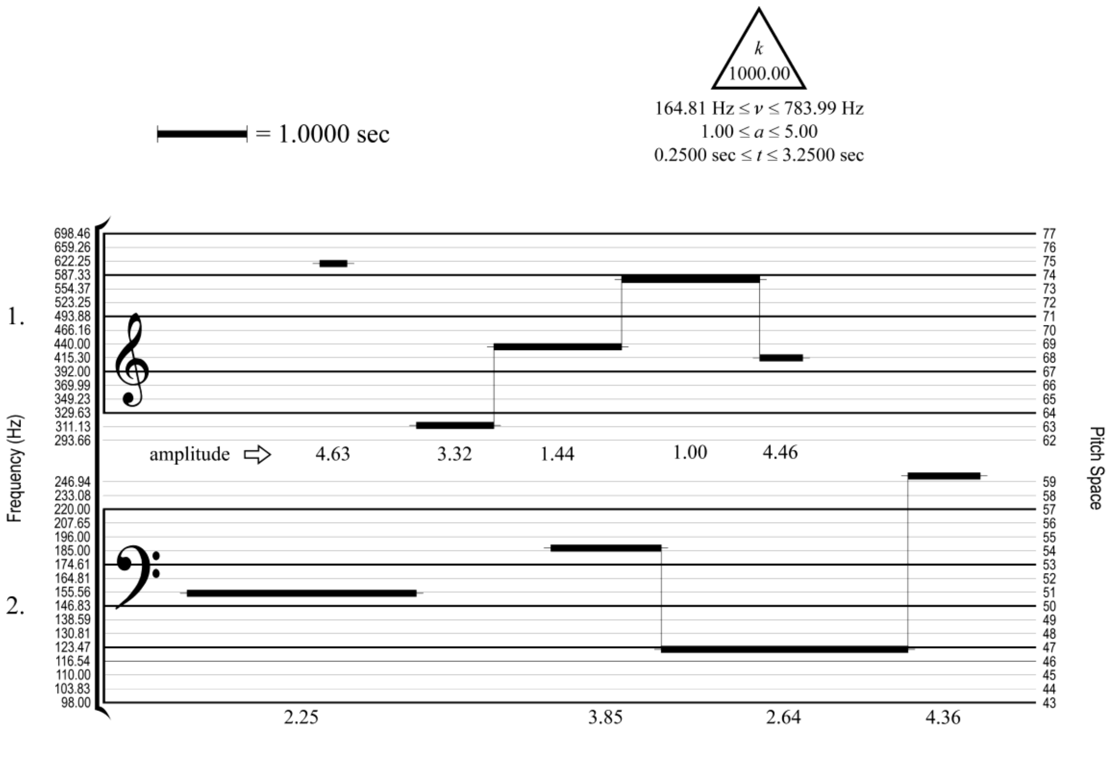


Figure 33. Contiguous Events in a State of Parametric Unity

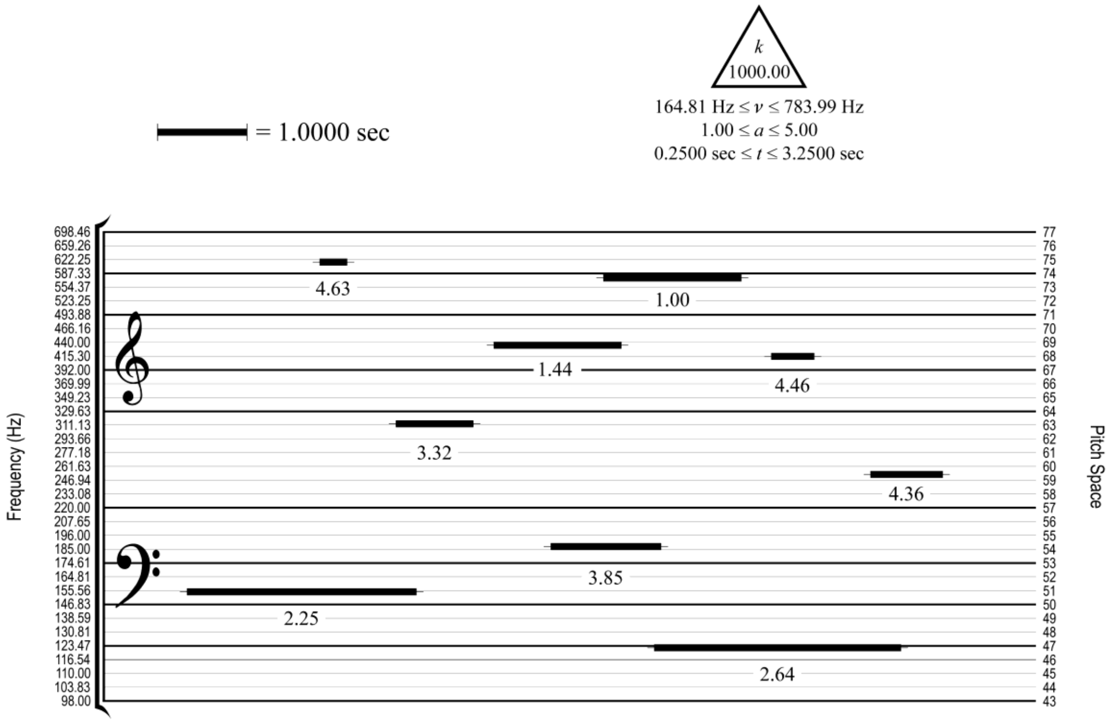


Figure 34. Non-Contiguous Events in a State of Parametric Unity

4.0 Observations and Conclusion

4.1 Observations

4.1.1 Serial Advancement

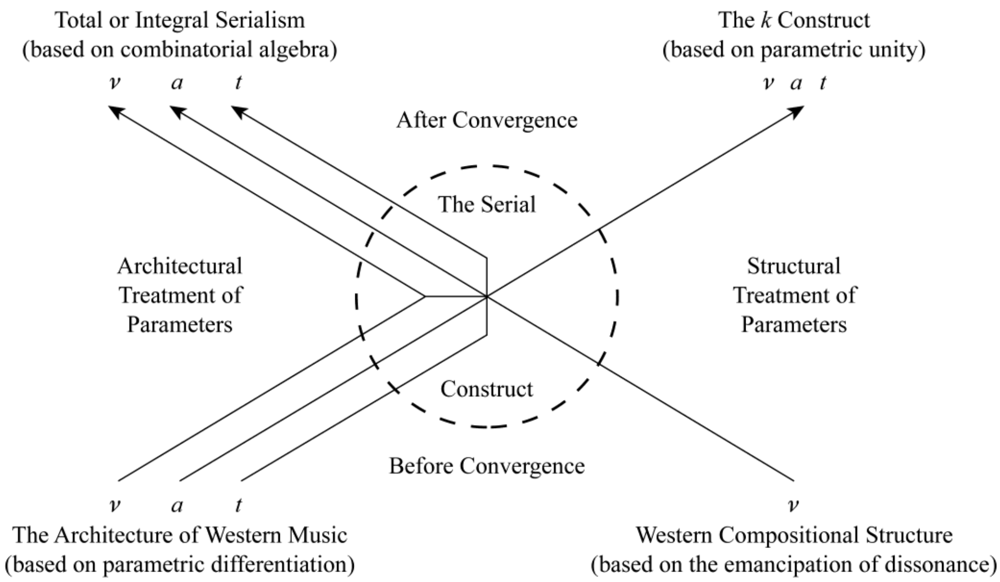


Figure 35. The Crossroads of Serialism

The convergence of Western compositional structure and musical architecture created two possible directions for serial advancement. At the *crossroads of serialism*, (see figure 35), the road taken was the architectural path of Total or Integral serialism, which applied the serial construct to diverse musical parameters in parallel by emulating the serial approach to pitch. The road not taken was the structural path of the *k* construct, which reintegrates the fundamental musical parameters (ironically) in series by balancing the inherent attributes of each musical event.

By localizing pitch-based relationships, Schoenberg ensured that any advancement beyond the serial construct would be transparametric in nature. However, the approach of Total or Integral serialism did not correlate with its underlying architecture. The inherent attributes of the chromatic set—its chromatic base, its cyclical repetition, its temperament—and the tonal relationships that resulted were unique to pitch and integral to the development of the tone row. The only aspect of the tone row that could be translated to amplitude and duration was its combinatorial algebra.

With the advent of serialism, Schoenberg identified a resonance between the emancipation of dissonance and the parametric differentiation of pitch, fulfilling the compositional potential inherent within the chromatic set. This understanding may have also prevented him from applying the serial construct to parameters that did not undergo a similar evolution or possess a similar resonance. While the *k* construct follows a solitary path leading from the crossroads of serialism, it also intersects with real or imaginary resonances in amplitude and duration that originate from outside the Western tradition.

4.1.2 Triangulated Stability

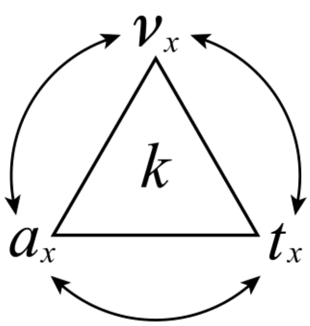


Figure 36. The Triangulated Stability of Parametric Unity

The *k* construct provides a form of *triangulated stability* wherein the pitch, amplitude and duration of a musical event rely upon each other for balance. The isolation of events afforded by this stability is not tantamount to structural chaos; it opens the possibility of a variable musical architecture that can be reinvented for and within each composition. However, such variability has a natural limit; just as the formal elimination of a tonal center can be viewed as a tonal center that shifts locally with each event, so too can the architecture shift locally and thus be formally eliminated. In this case, the triangulated stability of each event provides an audible basis for parametric unity as a fundamental reconception of music, without scales, without meter, without intervallic differentiation of any kind.

4.1.3 The Transverse Wave

The need to form a construct based on a single relationship within an isolated event prompted a search for a natural relationship between diverse sonic parameters. It was located in the average power P_{avg} of a transverse wave on a string, (see figure 37).

$$P_{avg} = 8\,\pi^3 R^2 V_0\,\rho_0\,\nu^2 a^2 t$$

Figure 37. The Average Power of a Transverse Wave on a String

In this relationship, R is the distance from the string to the listener, V_0 is the speed of sound, and ρ_0 is the density of air. These parameters typically remain constant during a musical performance, therefore, choosing a single value of P_{avg} isolates pitch, amplitude and duration, (see figure 38).

$$k = \nu^2 a^2 t$$

Where: $k = P_{avg} / 8\,\pi^3 R^2 V_0\,\rho_0$

Figure 38. Pitch, Amplitude and Duration in Terms of Average Power

While such a mixed-order equation could be used to generate musical events, small changes in pitch or amplitude would produce large changes in duration. By using only first-order variables $k = \nu_x\, a_x\, t_x$, the concept of uniform treatment inherent in both the tone row and the chromatic set can become manifest in a single event.

4.2 Conclusion

The boundary conditions of serialism point toward a new approach to structural organization as described by the *k* construct. Since each event formed by the construct contains only one dependent parameter—either pitch, amplitude or duration—the structural constraint is minimized. The result is the complete freedom of the two remaining parameters while maintaining an intrinsic sense of audible logic.

In terms of perception, the construct allows any parameter to range from the extremes of hearing to the limits of the 'just noticeable differences.' As stated, the construct accounts for the fundamental frequency of an event, and further possibilities exist in the control of harmonics via Fourier transformation. Events with values that change over time can be accounted for by integral calculus.

In terms of composition, the construct can be 'transposed' through the use of multiple *k* values. Since each pitch must be considered in the context of its amplitude and duration, such transpositions would apply to entire events. While the parametric balance of each event would always be maintained, the effect of the transposition would be analogous to a change in the perceived energy.

In terms of arrangement, while each event must have a positive duration in order to vibrate the air, compositions based on the *k* construct move neither forward nor backward in time. Without a temporal framework there is no concept of repetition, and the concept of the rest is changed from a non-event that is given the same weight in terms of duration as an event to a void that is equivalent to the space between movements or entire performances.

These conditions give rise to the premise that isolated events from disparate compositions—which by themselves function as compositions in miniature—share an affinity through common *k* values. Thus, in its ultimate application, the *k* construct provides a means of parametrically uniting not one composition but the whole of composition itself, since myriad threads of *k* values weave throughout all of sonic spectra and time.

5.0 References

1. Schoenberg, Arnold, *Hauer's Theories, 1923*, Style and Idea: Selected Writings of Arnold Schoenberg, Ed. Leonard Stein, University of California Press, 1975, 211.

2. Schoenberg, Arnold, *Composition with Twelve Tones (1), 1941*, Style and Idea: Selected Writings of Arnold Schoenberg, Ed. Leonard Stein, University of California Press, 1975, 218.

3. Ibid., 523:1. Schoenberg objected to the characterization of his technique as a "system" since it implied that consequences were inherent. While the pragmatic intent of his "method" was to explore new tonal resources—and in doing so, redefine "consequences" from a tonal standpoint—it imposed prospective limitations on tonal choice by design.

4. Ibid., 218-219. Schoenberg objected to comparisons between the tone row and the chromatic scale since the tone row did not perform a scalar function. The term "chromatic set" refers to the unordered set of tones equally differentiated by a chromatic base across all octaves.

5. Author Unknown, *Manuscript 1248, f º 138 vº*, Médiathèque Grand Troyes, Troyes, France. Used by permission.

6. Treitler, Leo, *The Early History of Music Writing in the West*, Journal of the American Musicological Society, Vol. 35, No. 2 (Summer, 1982), 237-279. See Treitler for a description of the functions of neumes.

7. Rufer, Josef, *Das Werk Arnold Schönbergs*, Kassel; New York: Bärenreiter, 1959.

8. Webern, Anton, *Concerto, Op. 24, No. I, 1, Basic Set*, Universal Edition AG, Wien, 1948. © 1948 Universal Edition AG, Wien / UE11830, © renewed, all rights reserved. Used by permission of European American Music Distributors LLC, US and Canadian agent for Universal Edition AG, Wien.